Modeling with Linear Algebra

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Instructor:

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Textbook:

Lay, David C.; Lay, Steven R.; McDonald, Judi J. (2016). *Linear Algebra and Its Applications*, 5th edition, Pearson.

Class meets:

Mo-Wed 11:00-12:30

Room G004

Gradings:

Midterm exam (25%), final exam (35%), 8 quizzes (15%), weekly homework (10%), a simulation project to be submitted/presented by the end of the course (15%)

Announcements

19 August - 21 August

1. **Uses of linear equations and matrices** (1st week)

19 August

Review of linear equations and matrix algebra (Section 1.1). Linear equations in economics, chemistry, and engineering (Section 1.6).

-Special assignment:

Write a short mathematical autobiography (check here for more info)

21 August

Linear equations in economics, chemistry, and engineering (Section 1.6). Adjacency matrix of a graph.

-**Homework** (due Monday, August 26, in class):

Section 1.1. Exercises 12, 16, 24, 26, 27.

Section 1.6. Exercises 3, 7, 13.

-Additional reading:

Wassily W. Leontief, Input-Output Economics, Scientific American, Vol. 185, No. 4 (October 1951), pp. 15-21

26 August - 28 August

1. **Uses of linear equations and matrices** (2nd week)

26 August

Invertible matrices (Section 2.2).

28 August

Linear algebra and cryptography.

-**Homework** (due on Wednesday, September 4, in class)

Write a proof of the fact that the (i,j)-entry of the kth power of the adjacency matrix of a graph counts the number of walks from vertex i to vertex j.

Section 2.2. Exercises 3, 6, 12, 33, 34, 41 (use the computer for the last one).

- Additional reading:

Francis Edward Su, Some Guidelines for Good Mathematical Writing, MAA Focus, August/September 2015, 20-22 or Pdf version

2 September - 4 September

1. **Uses of linear equations and matrices** (3rd week)

2 September

Interpolating polynomials. Matrix factorizations (Section 2.5).

1st Quiz held on Monday, September 2

4 September

Matrix factorizations (Section 2.5).

Homework (due on Wednesday, September 11, in class)

Section 1.2. Exercise 34 (use at least two different basis of polynomials, use the computer)

Section 2.5. Exercises 4, 10,12,17, 31, 32 (use the computer for the last two)

-Additional reading:

F. M. Dopico, Alan Turing and the origins of modern Gaussian elimination, Arbor, Vol.189-764 (2013) (read at least sections 1-3)

1st Quiz

9 September - 11 September

1. **Uses of linear equations and matrices** (4th week)

9 September

Determinant and its properties (Section 3.1 and 3.2).

11 September

Determinant and volume (Section 3.3)

2nd Quiz to be held September 11, in class

-Homework (due on Wednesday, September 18, in class)

-Section 3.1, Exercises 4, 41

-Section 3.2, Exercises 7, 33, 41, 44

-Section 3.3 , Exercises 5, 10, 17, 18, 27, 31

-Additional reading:

Arthur T. Benjamin and Naomi T. Cameron, Counting on Determinants, American Mathematical Monthly 112 (2005) 6

Arthur T. Benjamin & Gregory P. Dresden, A Combinatorial Proof of Vandermonde's Determinant, The American Mathematocal Monthly 114 (2007) 4, 338-341

Ira Gessel, Tournaments and Vandermonde determinants, Journal of Graph Theory 3 (1979) 3, 305-307

Who was Vandermonde?

2nd Quiz

16 September - 18 September

2. **Vector Spaces and linear transformations** (1st week)

16 September (holiday)

18 September

Vector spaces and subspaces (section 4.1). Spanning sets and linear independence. Basis and dimension of a vector space (Sections 4.3, 4.5). Coordinate systems (Section 4.4).

Homework (no homework this week)

23 September - 25 September

2. **Vector Spaces and linear transformations** (2nd week)

23 September

Linear transformations. Geometry of linear transformations and computer graphics. (sections 1.8, 1.9)

25 September

Geometry of linear transformations and computer graphics (sections 2.7). Kernel and image of a linear transformation. (Section 4.2)

-Additional reading:

Section 10.7 Windowing transformations - Hughes Van Dam McGuire Sklar Foley Feiner Akeley Computer graphics

30 September - 2 October

2. **Vector Spaces and linear transformations** (3rd week)

30 September

Kernel and image of a linear transformation. (Section 4.2)

2 October

The dimension theorem. (Section 4.5)

3rd Quiz

-Homework (due on Monday, October 7, in class)

-Section 1.9, Exercises 26

-Section 2.7, Exercises 9, 11, 19

-Section 4.1, Exercises 19, 20, 22

-Section 4.2, Exercises 31, 34, 36

-Section 4.3 , Exercises 20, 26, 34

3rd Quiz

7 October - 9 October

2. **Vector Spaces and linear transformations** (4th week)

7 October

Linear isomorphisms. The matrix of a linear transformation. (Section 1.9)

9 October

Change of basis and similarity. (Section 4.7)

-Homework (due on Monday, October 14, in class)

-Section 1.1, Exercise 18
-Section 1.2, Exercise 22
-Section 1.5, Exercise 26
-Section 1.6, Exercise 12
-Section 1.8, Exercise 9, 11, 20
-Section 2.2, Exercise 19
-Section 2.4, Exercise 33
-Section 2.9, Exercise 18
-Section 3.3, Exercise 25
-Section 46, Exercise 3
-Section 4.7, Exercises 6, 20 (use, for instance three basis from exercises 7 and 8 for part b.)

14 October - 16 October

Review of chapters 1 and 2 and Midterm

14 October

Review for Midterm: Quiz 4 Midterm practice exam

16 October

Midterm

Quiz 4 Midterm practice exam

Midterm

28 October - 30 October

3. **Inner product** (1st week)

28 October

Inner product and length. Ortoganility. Projections. (Section 6.1, 6.2, 6.3, and 6.7)

30 October

The Gram-Schmidt process. (Section 6.4)

-Homework (to be delivered on Monday, November 11 in class)

Section 6.1 Exercises 23,24,31
Section 6.2 Exercises 25,31,33, 34
Section 6.3 Exercises 2, 6, 12, 15, 23, 24
Section 6.4 Exercises 13, 16, 19, 20

4 November - 6 November

3. **Inner product** (2nd week)

4 November

Least-squares solutions. (Sections 6.5, and 6.6)

6 November

Inner product spaces (Section 6.7), Applications of inner products (Section 6.8)

5th quiz

Quiz 5

11 November - 13 November

4. **Eigenvalues and canonical forms** (1st week)

11 November

Eigenvalues and eigenvectors of a matrix. The characteristic equation. Diagonalization (Section 5.1, 5.2, 5.3, 5.5)

13 November

Diagonalization of symmetric matrices and the spectral theorem (Section 7.1). SVD Decomposition (Section 7.4)

Homework (due on Wednesday, November 20, in class)

Section 6.5 Exercises 1, 14, 17
Section 6.6 Exercises 1, 9
Section 6.7 Exercises 9, 14, 19, 21
Section 6.8 Exercises 5, 7

-Additional reading:

K. Pearson, On lines and planes of closest fit to systems of points in space, Philosophical Magazine 2 (1991) 559-572.

18 November - 20 November

4. **Eigenvalues and canonical forms** (2nd week)

18 November (holyday)

20 November

Principal Component Analysis (Section 7.5). The Page Rank algorithm. Markov chains (Section 10.1, 10.2).

-Homework (to be delivered on Monday, November 25 in class)

Section 6.8 Exercises 1, 3.
Section 7.1 Exercises 17, 24
Section 7.4 Exercises 13, 15
Section 7.5 Exercises 1, 2, 3, 4
Section 10.1 Exercise 18, 23
Section 10.2 Exercise 25

-Additional reading

P. Fernández Gallardo, Google's secret and Linear Algebra, EMS Newsletter 63, March 2007, 10-15.

K. Bryan, T. Leise, The $25,000,000,000 eigenvector, the linear algebra behind google, SIAM Review, August 2006.

Chapter 10 Linear Algebra and its applications

25 November - 27 November

4. **Eigenvalues and canonical forms** (3rd week)

25 November

Review Quiz 6 Practice for Final Exam

27 November

Final exam

Review Quiz 6 Practice for Final Exam

Final Exam MLA MSSG 19

Course project

Topics offered

Each student should choose one of the six following topics for his/her course project. Each student should write a 5-10 page easy about the topic and prepare a class presentation of about 30 minutes. Computer code or examples treated with a computer will give extra credit.

The references of the projects are just hints. Students are encouraged to do their own literature research about the chosen topic.

Discussion with the instructor and/or TA during the preparation of the project is highly recommended.

A-**Image compression**

Reference:

http://aix1.uottawa.ca/~jkhoury/haar.htm

B-**Facial recognition**

Reference:

http://aix1.uottawa.ca/~jkhoury/eigenfaces.htm

C-**Perspective rectification**

Reference:

Chapter 5, Section 12 of *Coding the Matrix: Linear Algebra Through Applications to Computer Science* by Philip N. Klein (http://codingthematrix.com/)

D-**Linear programming**

References:

http://aix1.uottawa.ca/~jkhoury/programming.htm
Chapter 13, Section 13 of *Coding the Matrix: Linear Algebra Through Applications to Computer Science* by Philip N. Klein (http://codingthematrix.com/)

E- **F_2-vector spaces (lights-out / perfect secret sharing)**

References:

Chapter 2, Section 8 of *Coding the Matrix: Linear Algebra Through Applications to Computer Science* by Philip N. Klein (http://codingthematrix.com/)
Marlow Anderson and Todd Feil, Turning Lights Out with Linear Algebra, Mathematics Magazine, Vol. 71, No. 4 (Oct., 1998), pp. 300-303 http://people.sc.fsu.edu/~jburkardt/classes/imps_2017/11_28/2690705.pdf
Vicente Muñoz, Number of solutions of the Ligths-out game, La Gaceta de la RSME Vol. 19, No. 1, 2016, pp. 83-97 http://agt.cie.uma.es/~vicente.munoz/lights-out.pdf
The mathematics of lights-out http://www.jaapsch.net/puzzles/lomath.htm

G- **Differential equations in the computer (e.g. Lotka-Volterra equations)**

Reference:

http://www.math.utah.edu/~gustafso/2250systems-de.pdf

List of applications discussed in the course

-Leontieff's closed and open input-output models (Introduction to chapter 1, Section 1.6, Section 2.6)

-Balancing chemical reactions (Section 1.6)

-Network flows and Kirchoff's and Ohm 's laws (Section 1.10)

-Discrete heat distributions (Exercises of Section 1.1)

-Constructing a Nutritious Weight-Loss Diet (Section 1.10)

-Difference equations and City-Suburban population models (Section 1.10)

-Mendelian genetics

-Adjacency graph of a matrix and Dominance-directed graphs

-Deflection of an elastic beam (Section 2.2)

-Vandermonde determinants and nonintersecting paths in a lattice

-Vandermonde determinants and tournaments

-Calculations of areas and volumes using determinants (Section 3.3).

-Using the determinant to calculate the change in volume or area induced by a linear transformation (Section 3.3).

-Applications to computer graphics: translations as multiplications by matrix via homogeneous coordinates, perspective projections (Section 2.7), and windowing transformations

-Elimination theory (existence of solutions of systems of two polynomial equations)

-Magic squares (there are no magic squares of size 2x2, construction of magic squares of size 3x3)

-Vector space of solutions of a linear difference equation (Section 4.8)

-Least-squares method, linear regression (Section 6.6)

-Trend analysis of data (Section 6.8)

-Principal components analysis (Section 7.5)

-Markov chains and page-rank algorithm (Section 10.1, 10.2)